A COMPARATIVE EVALUATION OF SOME TYPE-CURVE METHODS FOR THE DETERMINATION OF HYDROGEOLOGIC PARAMETERS OF FRACTURED ROCKS

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ABSTRACT

There are a limited number of drawdown data analysis methods for fractured rocks based on a constant extraction rate from a well for the determination of hydrogeologic parameters. The "double porosity" concept, which was proposed by Barenblatt et al. (1960), has been used extensively in the petroleum literature and for four decades it has been used for fractured rocks. In general, there are two approaches to problems of well test analysis which differ in the manner by which flow from a block to fissure is described. One approach assumes that flow occurs under pseudo-steady state conditions (Warren and Root, 1963) and the other approach assumes that flow occurs under fully transient conditions (Kazemi, 1969). To the knowledge of the author, the closed form analytical flow model of Streltsova (1976) is the first pseudo-steady state block-to-fissure flow model published in the ground water literature. However, Streltsova does not present a field example regarding the usage of her model to determine rock hydraulic parameters. By using the general fractured rock modeling methodology of Barenblatt et al. (1960), Moench (1984) extended the Warren and Root (1963) and Kazemi (1969) modeling approaches to water flow in fractured rocks and generated some analytical models in the Laplace domain and took their inverse Laplace transform using the Stehfest (1970) algorithm. The purpose of this paper is to make a comparative evaluation between Streltsova's (1976) pseudo-steady state block-to-fissure flow model and Moench's (1984) pseudo-steady state block-to-fissure and transient block-to-fissure flow models. For this purpose, the drawdown data set in Moench (1984) for the Nevada Test Site is analyzed using the type-curves generated from the models of Streltsova and Moench. The results of the type-curve methods showed that the hydraulic parameters determined from these different models are significantly close to each other despite the fact that the mathematical approaches of the models of Streltsova and Moench are different. As a result, it is recommended that both the model of Streltsova and the models of Moench be used as alternative models for the hydrogeologic parameter determination of fractured rocks.

INTRODUCTION

The first quantitative theoretical study for the fluid flow in fractured rocks was carried out by Barenblatt et al. (1960) by introducing the "double porosity" concept to the aquifer hydraulics literature. This concept is being used extensively both in the petroleum and aquifer literatures. In general, there are two approaches for problems of fractured rock aquifers which are composed of fissures and blocks or matrices. The first approach assumes that flow occurs under pseudo-steady state conditions (Warren and Root, 1963). The second approach assumes that flow occurs under fully transient conditions (Kazemi, 1969). The pseudo-steady state block-to-fissure flow assumption provides mathematical simplicity in solving the problem.

The purpose of this paper is to make a comparative evaluation between Streltsova's (1976) pseudo-steady state block-to-fissure flow model and Moench's (1984) pseudo-steady state block-to-fissure and transient-block-to fissure flow models.

PSEUDO-STEADY STATE BLOCK-TO-FISSURE FLOW MODELS

Barenblatt, Zheltov, and Kochina (BZK) Model

The hydrodynamic foundations of flow through fractured rocks were established by Barenblatt and Zheltov (1960) and Barenblatt, Zheltov, and Kochina (1960) by considering overlapping continuous porous and fissured media. The authors assumed that the flow domain formed by these two media represent a fractured rock formation, which consists of an extensive system of randomly distributed and arbitrarily oriented fissures in a rock having primary porosity. Therefore, in the vicinity of each point there are two pressure values. One of them is an average pore fluid pressure of the porous medium and the other one is an average fissure pressure. In the BZK model; to have the representative average pressure values, the authors assumed an elemental volume characterizing the properties of medium to be the size of a sufficiently large number of porous blocks of a network of fractures. A medium is called "double porosity medium" when it is controlled by the fracture and bulk hydraulic properties of the same order of magnitude. A schematic representation of a double porosity medium is shown in Figure 1. Based on the BZK modeling concepts for a fractured medium, two models regarding flow created by a well in a double porosity medium are described below. There are some fundamental differences between these two models. Therefore, these models will be presented by keeping the authors' notational conventions.

Streltsova's Model

Fissure Flow Governing Equations and Drawdown Solution: By applying the equation of continuity, Streltsova (1976) obtained the general differential equation for the fissure flow under confined flow conditions as

$$T\left(\frac{\partial^2 s_f}{\partial r^2}+\frac{1}{r}\frac{\partial s_f}{\partial r}\right)=S^*\frac{\partial s_f}{\partial t}+S\frac{\partial s_p}{\partial t} \qquad (1)$$

where $T(L^2T^{-1})$ is the aquifer transmissivity; s_f (L) the drawdown of hydraulic head at any point of fissure portion of aquifer; s_p (L) the drawdown of hydraulic head at any point of porous portion of aquifer; r (L) radial distance; t (T) time; S* (unitless) the storage coefficient of fissured space; and S (unitless) the storage coefficient of porous space. According to the modified form of Warren and Root (1963) to the BZK model, Streltsova (1976) wrote the following equation:

$$S\frac{\partial s_p}{\partial t} = k\frac{s_f - s_p}{l} = p(s_f - s_p) \qquad (2)$$

where k is the hydraulic conductivity (LT^{-1}) of the porous block, l (L) is the average block dimension taken as the distance between the center and the surface of the block, and p (T^{-1}) is the parameter of the fissure-pore flow exchange. By solving Eq. (2) with the corresponding initial and boundary conditions, Streltsova (1976) obtained the following integro-differential equation:

$$T(\frac{\partial^2 s_s}{\partial r^2} + \frac{1}{r}\frac{\partial s_f}{\partial r}) = S*\frac{\partial s_f}{\partial t} + p\int_0^t e^{-\frac{p}{S}(t-\tau)}\frac{\partial s_f}{\partial t}d\tau \qquad (3)$$

Streltsova (1976, page 407) states that because the storage properties of fractured aquifer are assumed to be dependent on the primary porosity, the water pumped from the aquifer is the water released from the porous blocks. Therefore, the boundary condition at the well is

$$-\frac{\partial s_p}{\partial r} = \frac{Q}{2\pi T} \quad r \to 0 \qquad (4)$$

By using Eq. (2), Streltsova (1976) expressed s_p in terms of s_f and she modified the boundary condition given by Eq. (4) as

$$-\frac{\partial s_f}{\partial r} = \frac{Q}{2\pi T}(1 - e^{-\frac{p}{S}t}) \qquad r \to 0 \qquad (5)$$

Streltsova (1976) obtained the fissure flow drawdown distribution from Eq. (3) with the boundary condition at the well given by Eq. (5) as

$$s_f = \frac{Q}{4\pi T}W_f \qquad (6)$$

and presented a table for the calculated values of W_f for different values of η and r/B as a function of

$$\theta = \frac{4Tt}{r^2 S} \qquad (7)$$

Details of the function of W_f are given in Streltsova (1976). For η = 100 the type curves are presented in Figures 2.

Porous Block Flow Governing Equations and Drawdown Solution: The general integro-differential equation for the flow in porous blocks is identical to Eq. (3), except for the symbols s_p replacing s_f. Streltsova (1976) obtained the block flow drawdown distribution from Eq. (3) with the boundary condition at the well given by Eq. (4) as

$$s_p = \frac{Q}{4\pi T}W_p \qquad (8)$$

Details of the function of W_p are given in Streltsova (1976). For η = 100 the type curves are presented in Figure 5.

Some Clarifying Points Regarding Streltsova (1976) Model: Comparison of Streltsova's fissure-flow type curves (Streltsova, 1976, p. 408, Figure 2) with Moench's fissure-type curves (Moench, 1984, pp. 840-841, Figures 11 through 13) show that they have different shapes. The author of this paper brought this point as well as the type-curve analysis results to the attention of Streltsova and exchanged e-mail messages (T.D. Streltsova, personal communication, January 7, 2007). Streltsova's response is as follows: "After publication of the 1976 *Water Resources Research* paper, I made a correction that "porous block flow" equation in the paper is the "fissure flow" equation and, alternatively, the "fissure flow" in the paper is the "porous block" flow equation. The porous block solution is relevant to the low-permeability media and is appropriate to use, for example, in stratified reservoirs with cross flow. So, you are very right in your comments and the analysis." Therefore, Eq. (6) is the "porous block flow" equation and s_f and W_f should be replaced by s_p and W_p, respectively. Likewise, Eq. (8) is the "fissure flow" equation and s_p and W_p should be replaced by s_f and W_f, respectively.

Moench's Model

Governing Equations for Fissure and Block Flows: Moench (1984) used the general model of BZK for block-to-fissure flow with slightly different notation of Streltsova (1976). Moench assumed that the controlling differential equation for flow in the fissure network is described by the familiar saturated ground water flow equation with a source term to account for contributions from the matrix rock in the cylindrical coordinates as

$$K\left(\frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}\right)=S_s\frac{\partial h}{\partial t}+q_\alpha \tag{9}$$

where K (LT^{-1}) is the hydraulic of the fissure system; h (L) is the hydraulic head in fissure; r (L) is the radial distance measured from the center of pumped well; S_s (L^{-1}) is the specific storage of the fissure system; q_α (T^{-1}) is the source term for pseudo-steady state block-to-fissure flow; and t (T) is the time. For the matrix system, the corresponding differential equation is

$$K'\left(\frac{\partial^2 h'}{\partial r^2}+\frac{1}{r}\frac{\partial h'}{\partial r}\right)=S'_s\frac{\partial h'}{\partial t}-q_\alpha \tag{10}$$

where K' (LT^{-1}) is the hydraulic conductivity of the block system; h' (L) is the hydraulic head in block; and S'_s (L^{-1}) is the specific storage of the block system. The assumption is that water moves locally from matrix to fracture, but not from matrix to fracture. Namely, the matrix is the source of water. Therefore, Eq. (10), by neglecting its left-hand side terms, can be written as

$$S'_s\frac{\partial h'}{\partial t}-q_\alpha=0 \tag{11}$$

In the steady state assumption of the BZK model (Barenblatt et al., 1960), flow from matrix to fracture is assumed to be proportional to the difference in head between the two systems. In other words,

$$q_\alpha=-\alpha K'(h'-h) \tag{12}$$

where α (L^{-2}) is a parameter that depends on the geometry of the matrix blocks. Note that the combination of Eqs. (11) and (12) is equivalent to Eq. (2). If the initial head in both the fissure and block is h_0, then the drawdowns can be defined as s = h_0 – h and s' = h_0 – h' and from the combination of Eqs. (9) and (12) can be written as

$$K\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)+\alpha K'(s'-s) = S_s\frac{\partial s}{\partial t} \quad (13)$$

It is important to note that the term αK' is always together as a product. The initial conditions can be expressed as

$$s = s' = 0 \quad at \quad t = 0 \quad (14)$$

At a large distance

$$\lim_{r\to\infty} s = \lim_{r\to\infty} s' = 0 \quad (15)$$

At the well

$$\lim_{r\to 0}\left(r\frac{\partial h}{\partial r}\right) = -\frac{Q}{2\pi KH} \quad (16)$$

Where H (L) is the formation thickness and Q (L^3T^{-1}) is total well discharge. Eq. (16) is based on the assumption that the extracted water enters the well through fractures. This is a reasonable assumption as long as the well actually intersects fractures, because the matrix is assumed to have considerable lower hydraulic conductivity (Moench, 1984).

Solution in the Laplace Domain: Moench (1984) solved the above defined problem [Eqs. (13) through (16)] in the Laplace domain and obtained the following solution:

$$\overline{h_D} = \frac{2}{p}K_0[r_D(p+\overline{q_D})^{\frac{1}{2}}] \quad (17)$$

where p is the Laplace transform variable and

$$\overline{q_D} = \frac{p}{\frac{1}{\sigma}+\frac{p}{\lambda}} \quad (18)$$

and

$$\lambda = \alpha\frac{K'}{K}r^2_w \qquad r_D = \frac{r}{r_w} \qquad \sigma = \frac{S'_s}{S_s} \quad (19)$$

and K_0 is the modified Bessel function of the second kind and zero order. Moench (1984) took the inverse Laplace transform of Eq. (17) by using the Stehfest (1970) algorithm in his FORTRAN computer program. This algorithm has proven to be exceptionally accurate for a variety of aquifer and petroleum engineering problems. Moench and Ogata (1984) provided the foundations of this algorithm for some aquifer problems with applications.

A Comparative Evaluation of the Streltsova (1976) and Moench (1984) Pseudo-Steady State Block-to-Fissure Flow Models: In the previous sections, the foundations of Streltsova (1976) and Moench (1984) pseudo-steady state block-to-fissure flow models are presented. A comparative evaluation and some important points of these two models are as follows: (1) Both Streltsova and Moench use the BZK (Barenblatt et al., 1960) and Warren and Root (1963) approach; but, they use different solution techniques in solving the corresponding boundary-value problems; (2) Streltsova uses two different boundary conditions at the well, one for block flow with standard form and one for block flow with complex form; and they are given by Eqs. (4) and (5), respectively. However, Moench uses the standard form of boundary condition at the well, Eq. (16), on the assumption that the extracted water enters the well through fractures. In other words, Moench's model neglects the contribution from the blocks or matrices. Regarding this assumption Moench (1976, p. 832) states that this is generally a reasonable assumption as long as the well actually intersects fractures, because the matrix is assumed to have considerable lower hydraulic conductivity; and (3) in terms of drawdown, Eq. (12) of Moench can be expressed as

$$q_\alpha = -\alpha K'(h' - h) = \alpha K'(s' - s) \tag{20}$$

Substitution of Eq. (20) into Eq. (11) and multiplying both sides by H gives

$$S'\frac{\partial s'}{\partial t} = \alpha K' H(s - s') \tag{21}$$

where S' = S'_sH is the storage coefficient of the matrix (Streltsova calls it S). Eq. (21) is the equivalent form of Streltsova, which is given by Eq. (2). Therefore, Streltsova's p is

$$p = \alpha K' H \tag{22}$$

The dimensions of p is T^{-1} because the dimensions of α, K', and H are L^{-2}, L, and LT^{-1}, respectively.

TRANSIENT BLOCK-TO-FISSURE FLOW MODELS

In order to account for transient flow from blocks-to-fissures, it is necessary to specify the block geometry. Kazemi (1969) assumed transient block-to-fissure flow and used a finite-difference model for simulation of flow to a well. Kazemi assumed that the fractured rock mass can be simplified as having slabs of blocks and fissures. The thickness of the blocks and the aperture of the fissures represent average fracture spacing and aperture. Moench (1984) applied a similar approach to a fractured rock aquifer having slab-shaped or sphere-shaped blocks as shown in Figure 3. Moench solved the corresponding boundary-value problem in the Laplace domain and obtained a solution in the form of Eq. (17) with expressions for q_D [Moench, 1984; Eq. (15) for slab-shaped blocks; Eq. (18) for sphere-shaped blocks]. Likewise, Moench also solved the same problem with the skin effects and obtained a solution in the form of Eq. (17) with different expressions for q_D [Moench, 1984; Eq. (46) for slab-shaped blocks; Eq. (53) for sphere-shaped blocks]. All of the derivational details of these solutions are included in Moench (1984).

COMPARISON OF STRELTSOVA'S AND MOENCH'S MODELS

Description of the Pumping Test

The drawdown data is taken from Moench (1984, Table 2), which correspond to a pumping test in the vicinity of Yucca Mountain at the Nevada Test Site. Core samples showed that most of the fractures are steeply dipping and coated with deposits of silica, manganese and iron oxides, and calcite. According to description of Moench, there are five major zones of entry over a depth interval of about 400 m, which is taken to be the reservoir thickness (H). The test was pumped at a constant rate of Q = 35.8 L/s for nearly 3 days. The drawdown versus time curve at an observation well (r = 110 m from the pumped well) is shown in Figure 4, which represents fissure flow.

Type-Curve Analysis With the Pseudo-Steady State Block-to-Fissure Flow Models of Streltsova and Moench

Using Streltsova's type-curve data for fissure flow (Streltsova, 1976, p. 410, Table 2), three sets of type curves were generated for η = 10, 100, and 1,000. Of these type curves, r / B = 0.2 in the family of η = 100 established the best match with the observed data (see Figure 5). In the same figure the type-curve matching coordinates are also shown. The results for K, S_s, S'_s, and $\alpha K'$ calculated from the type-curve matching data are included in Table 1.

Moench (1984) does not present results from his pseudo-steady state block-to-fissure flow model. Type-curves were generated by running the FORTRAN computer program called DP_LAQ developed by Moench (1984). By trial-and-error, it was found that the type-curve corresponding to r_D = 1,000, σ = 200, and λ = 2.0E-06 established the best match with the observed data (Figure 6). The type curves for r_D = 100 and λ = 2.0E-06 with three different σ values (2.0E+01, 2.0E+02, and 2.0E+03) indicated that no-type curve matching is possible for these parameters. The results for K, S_s, S'_s, and $\alpha K'$ calculated from the type-curve matching data are included in Table 1. As can be seen from Table 1, the K, S_s, and S'_s values derived from Streltsova and Moench's models compare favorably; there is approximately 1.5 orders of magnitude difference of the $\alpha K'$ values.

Table 1. Comparison of the hydrogeologic parameters determined from the type-curve matching results of Moench and Streltsova models.

Parameter	Moench's Transient Model	Moench's Pseudo-Steady State Model	Streltsova's Pseudo-Steady State Model
K (m/s)	1.2E-05	7.8E-06	1.4E-05
S_s (m^{-1})	1.7E-06	1.5E-06	2.3E-06
K_s / b_s (s^{-1})	1.8E-09	-	-
K' (m/s)	7.2E-08	-	-
S'_s (m^{-1})	3.4E-04	3.0E-04	2.3E-04
$\alpha K'$ ($m^{-1}s^{-1}$)	-	1.9E-09	4.7E-11

Table 2. Moench's transient block-to-fissure flow example results and reanalysis results.

Parameter	Moench's Results	Reanalysis Results
K (m/s)	1.0E-05	1.2E-05
S_s (m^{-1})	1.5E-06	1.7E-06
K_s / b_s (s^{-1})	5.0E-08	1.8E-09
K' (m/s)	2.0E-06	7.2E-08
S'_s (m^{-1})	3.0E-04	3.4E-04

Type-Curve Analysis With the Transient Block-to-Fissure Flow Model of Moench and Comparison With the Other Models

Type-curves were generated by running the FORTRAN computer program called DP_LAQ developed by Moench (1984). By trial-and-error, it was found that the type-curve corresponding to r_D = 1,000, W_D = 1,000, σ = 200, S_w = 0, γ = 1.342E-03, and S_F = 1 established the best match with the observed data (Figure 7). Table 2 presents the type-curve results of Moench (1984) and the results of reanalysis for K, S_s, K_s / b_s, K', and S_s. The results are close with the exception of K' values. In Table 3, the results are compared with Moench's pseudo-steady state block-to-fissure flow model for K, S_s, and S'_s values, which compare favorably. In Table 1, Moench's transient block-to-fissure flow model results are also compared Moench's pseudo-steady state block-to-fissure flow model results for K, S_s, and S'_s values, which compare favorably.

Table 3. Comparison of the hydrogeologic parameters determined from the type-curve matching results of Moench's models.

Parameter	Moench's Transient Model	Moench's Pseudo-Steady State Model
K (m/s)	1.2E-05	7.8E-06
S_s (m^{-1})	1.7E-06	1.5E-06
K_s / b_s (s^{-1})	1.8E-09	-
K' (m/s)	7.2E-08	-
S'_s (m^{-1})	3.4E-04	3.4E-04
αK'	-	1.3E-09

CONCLUSIONS

The conclusions drawn from this work can be summarized as follows: (1) Both the pseudo-steady state block-to-fissure flow models of Streltsova (1976) and Moench (1984) for K, S_s, and S'_s produced significantly close results. However, there is a 1.5 orders of magnitude between their αK' values; (2) Moench's (1984) transient block-to-fissure flow model and the pseudo-steady state block-to-fissure flow models of Streltsova (1976) and Moench (1984) produced significantly close results for K, S_s, and S'_s; (3) The only advantage of the transient block-to-fissure flow model is that separate values for α and K' values can be determined; whereas with the pseudo-steady state block-to-fissure flow model only the value of the product αK' can be determined; (4) The double porosity and slab-shaped modeling approaches for fractured media produced significantly close hydrogeologic parameters; and (5) The above-given conclusions are based on a data set only. Additional comparisons are recommended by using different data sets for fractured media.

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BIOGRAPHY

Vedat Batu, Ph.D., PE, is currently with URS Corporation in Chicago, Illinois, USA and is a ground water hydrologist with more than three decades of experience. His areas of expertise includes flow and solute transport in surface and subsurface flow systems, mathematical modeling, research and practices in these fields. He is the author of many publications in internationally-recognized journals such as *Ground Water*, *Journal of Hydrology*, *Journal of Hydraulic Engineering*, *Water Resources Research*, and *Soil Science Society of America Journal*. He is also the author of two books titled *Aquifer Hydraulics: A Comprehensive Guide to Hydrogeologic Data Analysis* (John Wiley & Sons, Inc., New York, New York, 727 pages, 1998) and *Applied Flow and Solute Transport Modeling in Aquifers: Fundamental Principles and Analytical and Numerical Methods* (CRC Press Taylor & Francis Group, Boca Raton, Florida, 667 pages, 2006.

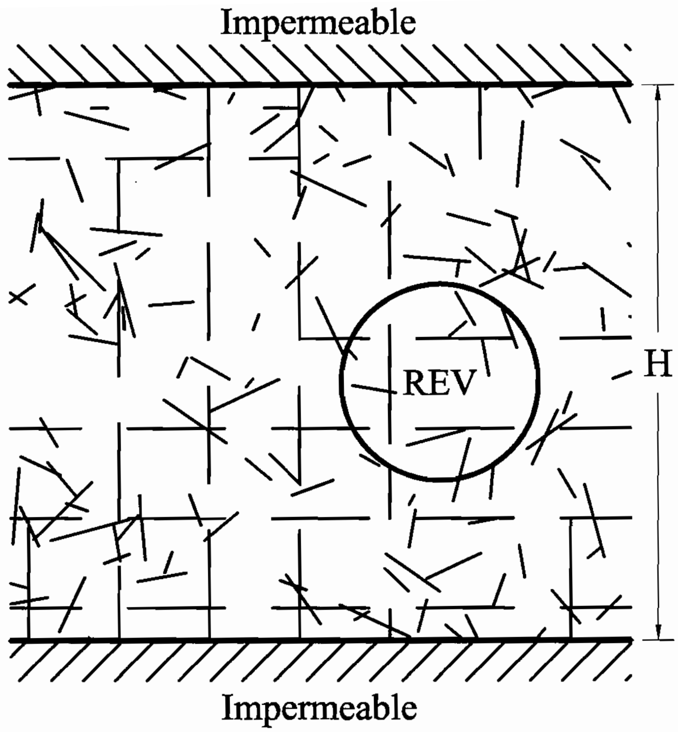

Figure 1. Schematic representation of a double-porosity fractured aquifer of thickness H showing a typical representative elementary volume (REV) (Adapted from Moench, 1984).

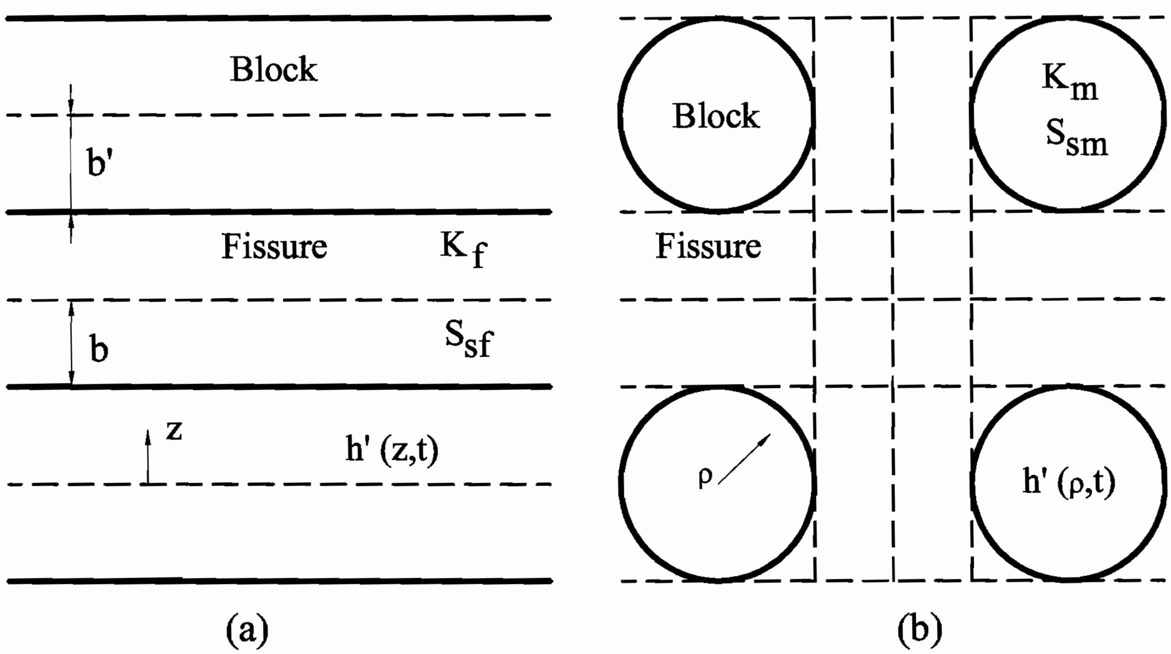

Figure 3. Geometrical configuration for (a) slab-shaped blocks and (b) sphere-shaped blocks (Adapted from Moench, 1984).

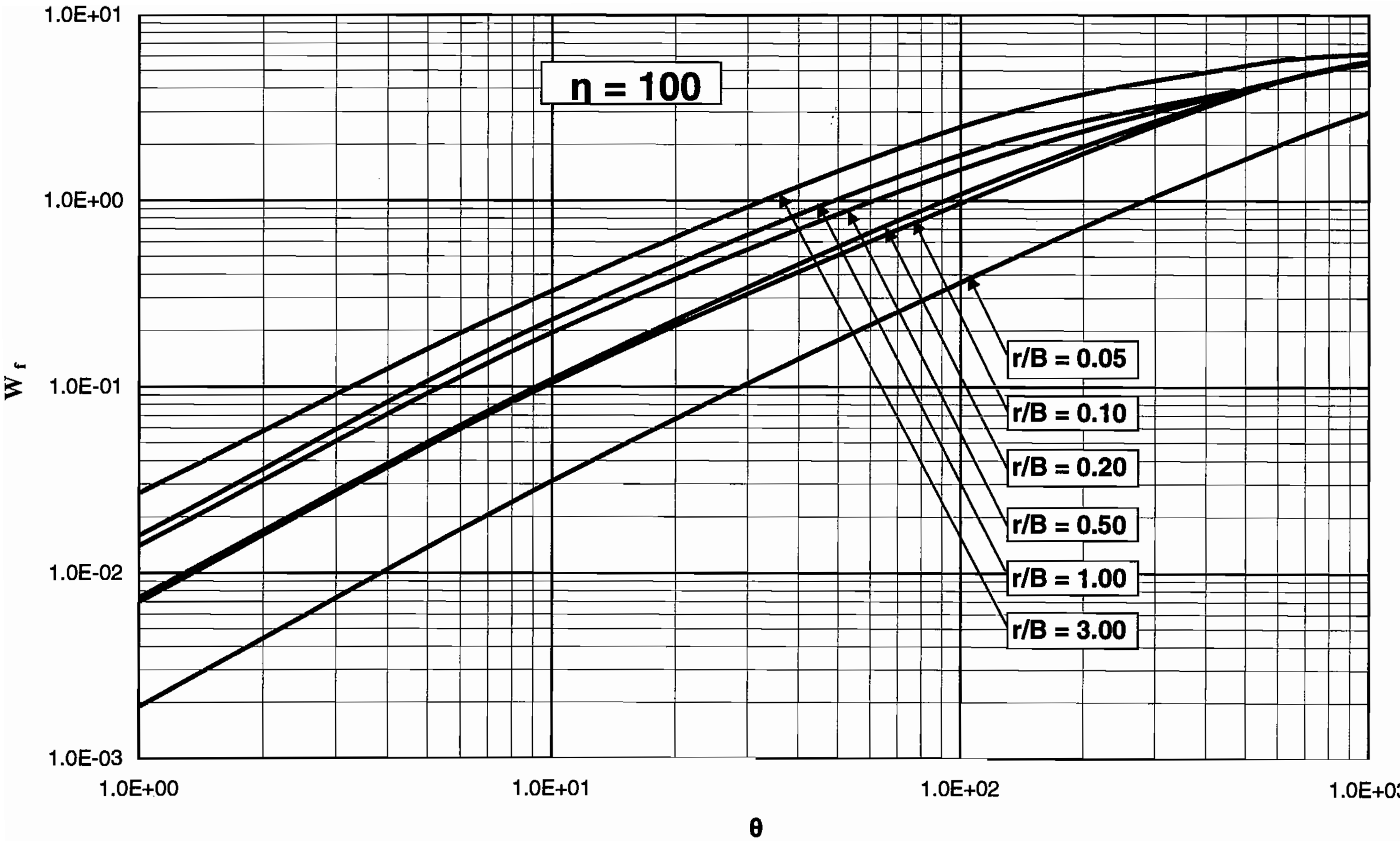

Figure 2. Type curves for fissure flow for $\eta = 100$ based on Streltsova (1976) model.

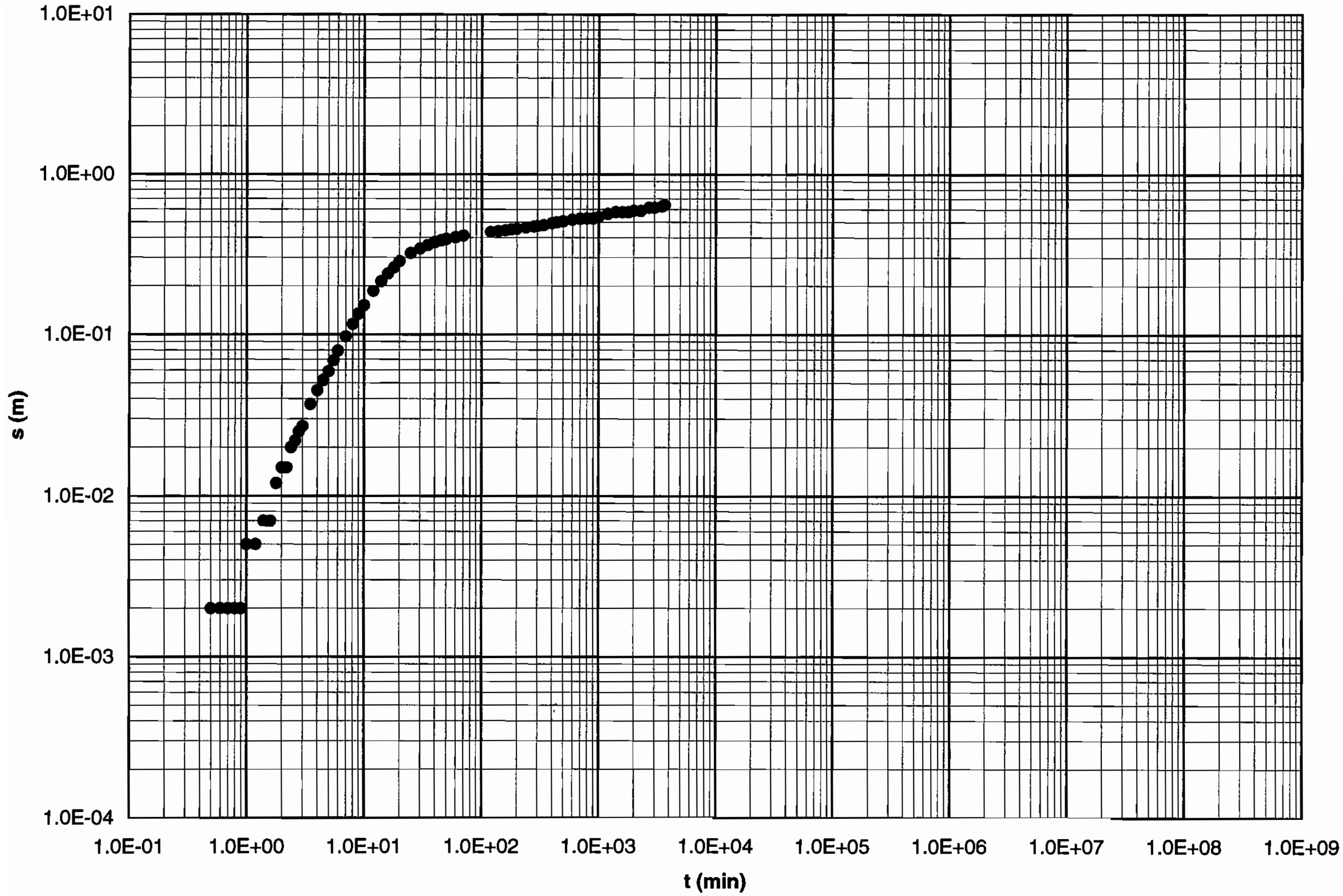

Figure 4. Drawdown data for the examples (Moench, 1984).

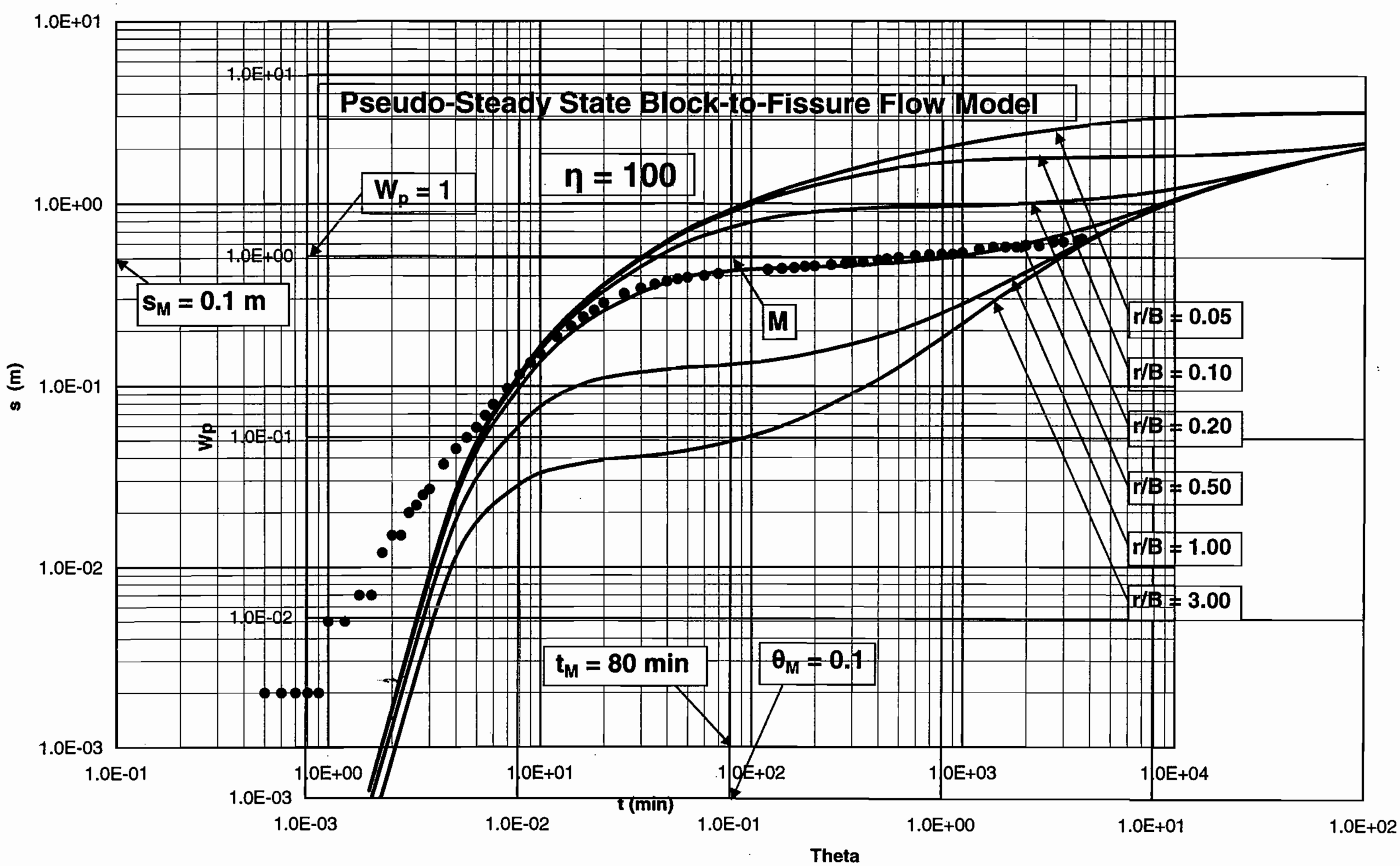

Figure 5. Type-curve matching with Streltsova (1976) pseudo-steady state block-to-fissure-flow model.

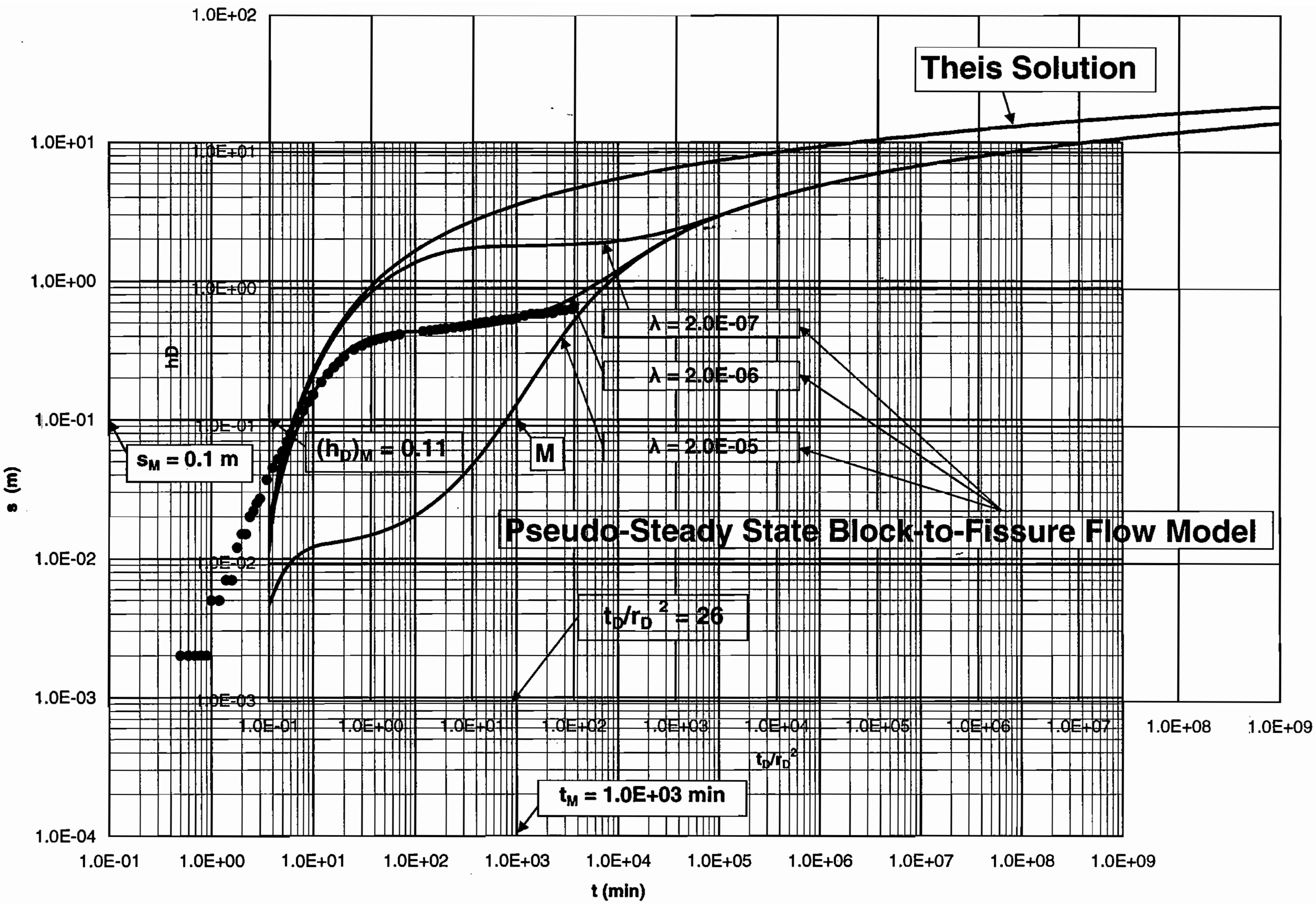

Figure 6. Type-curve matching with Moench (1984) pseudo-steady state block-to-fissure-flow model.

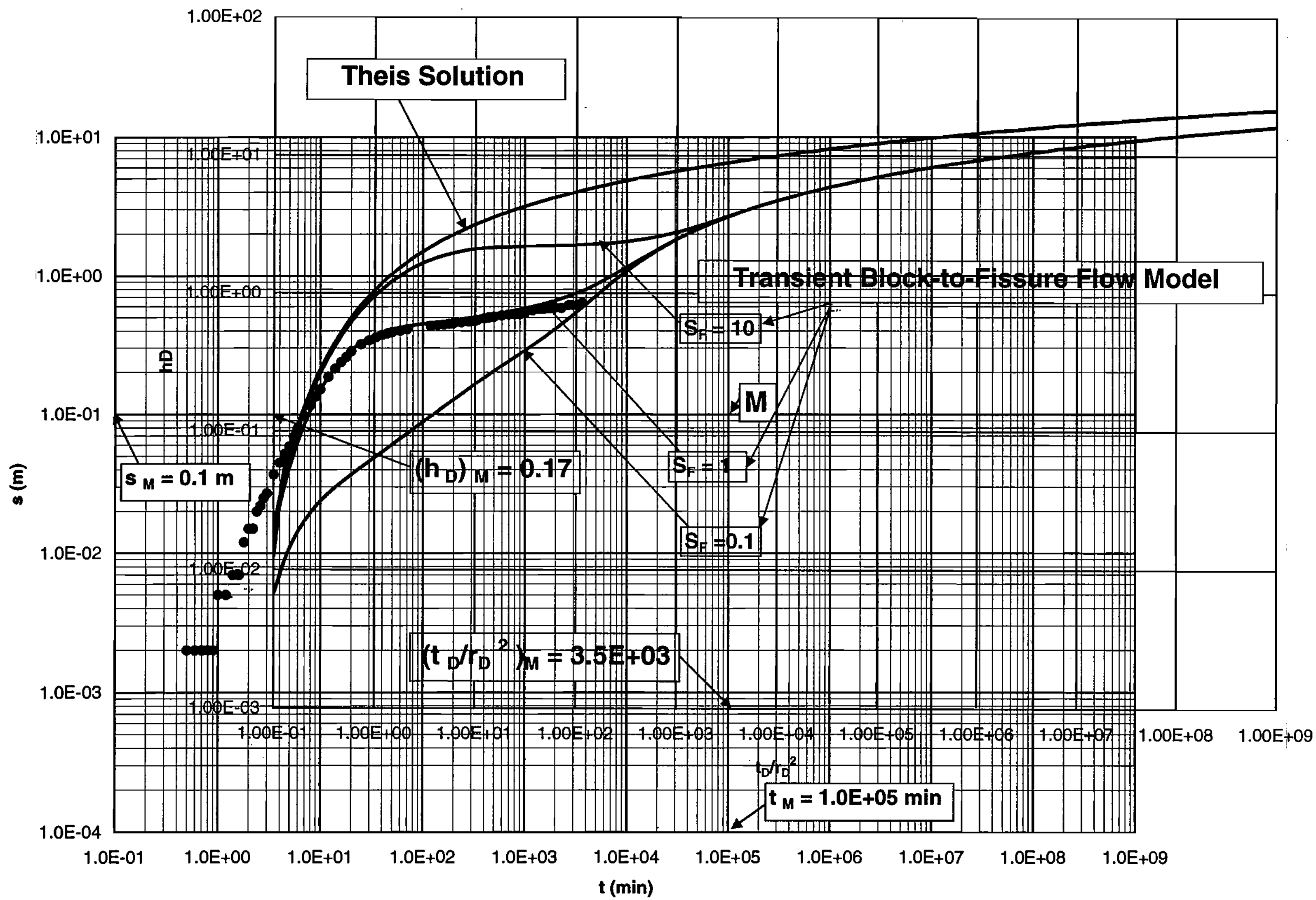

Figure 7. Type-curve matching with Moench (1984) transient-block-to-fissure flow model.