Assessment of Hydraulic Capture Through Interpolation of Measured Water Level Data

Matthew Tonkin\(^1,2\), Steven Larson\(^1\) and Chris Muffels\(^1\)

2. University of Queensland, Australia.

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Outline of This Presentation

- A Problem Statement
- Some History and Motivation
- A brief introduction to Kriging
- Some Reality Checks to be mindful of
- The inclusion of Particle Tracking
- A single illustrative Example
Background and Developments

• The basis of the approach described here – combining kriging and hydrology – is an area of extensive historical investigation and literature

• This presentation discusses one fairly recent development in particular

• David Dougherty will discuss extensions to the approach described here that he has developed and applied
Problem Statement

- How do we improve the inference that can be drawn from a water level data set?
History

- Rapid development of several large-scale, multi-well pump-and-treat (P&T) remedies.
- Various options for returning treated water including injection, infiltration.
- Quarterly monitoring reports.
- Regular meetings.
History
History

• Common questions asked at these meetings.....
  
  o *How is the system performing?*
  o *Is capture developing?*

• The contents of quarterly reports were not intended to include capture zones.
Making Maps

• Water levels are measured at a lot of wells - *but*,

• It is still necessary to interpolate between measured data in order to make maps - *and*,

• Most maps we make of water levels don’t convey much information.
Making Maps

• If we acknowledge that in order to make maps we must perform some form of interpolation - then,

• the question is which of the available methods is the most suitable for our needs?

  - inverse distance, kriging, radial basis functions, others?
Motivation

- To increase the inference that can be drawn from water levels measured in monitoring wells

in order to

- Assist with inferring pump-and-treat system performance.
Introduction to Kriging

- Kriging is a method for interpolating from known data to intermediate locations.
- When rigorously implemented, interpolation weights are determined from the data.
- Where there is assumed to be no measurement error, and there are no replicates, kriging is an exact interpolator.
- Under certain assumptions about spatial statistics (variogram), kriging provides an estimate of the variance in the estimation error.
Some Bases for Kriging

- The variogram - intuitively, a descriptor for how information diminishes with separation distance.
- Linear algebra - a method for solving the matrix equations to estimate a point.

Variography is fun - but can be taken too seriously .... data sets usually inadequate
Simple or Ordinary Kriging

- Estimated value is a function of the surrounding, known data
- The weight given to the known data is proportional only to the separation distance of each known point from the estimation point

\[ Z_{est} = f(R) \]

1. Assumptions about the mean of the quantify being estimated vary – for simple kriging, zero; for ordinary kriging, data average
Kriging With a Trend\(^1\)

\[ Z_{est} = A + BX + CY \]

A = ‘offset’ term
B = gradient in the X direction
C = gradient in the Y direction

Trend coefficients must be linear

\[ Z_{est} = f(X, Y, R) \]

1. Often termed ‘Universal Kriging’
Kriging With a Trend

- Kriging with linear trend has been shown to be effective in aquifer systems (e.g. Ogallala) where regional patterns are dominant.

- However - where we have singularities - such as wells - severe local departures from this linear trend occur.
The Cooper-Jacob Equation

\[ S = \frac{Q}{4\pi T} \ln \left( \frac{2.25Tt}{r^2S} \right) \]

s = drawdown
Q = pumping rate
T = transmissivity
S = storage
r = separation distance
t = time
The Cooper-Jacob Equation

\[ S = \frac{Q}{4\pi T} \ln \left( \frac{2.25 T t}{r^2 S} \right) \]

\( s \) = drawdown
\( Q \) = pumping rate
\( T \) = transmissivity
\( S \) = storage
\( r \) = separation distance
\( t \) = time

Note - this is a function of time - but we wish to deal with steady state.
The Cooper-Jacob Equation

\[ S = \frac{Q}{4\pi T} \left( \ln \left( \frac{2.25Tt}{S} \right) + \ln \left( \frac{1}{r^2} \right) \right) \]

This is a function of time

This is not
The Cooper-Jacob Equation

\[ s = \frac{Q}{2\pi T} \ln(r) + \text{offset} \]
The Cooper-Jacob Equation

\[ s = \frac{1}{2\pi T} Q \ln(r) + \text{offset} \]
The Cooper-Jacob Equation

\[ s = \frac{1}{2\pi T} Q \ln(r) + \text{offset} \]

\( \text{Log trend coefficient} \)
Multiple Extraction Wells

\[ s_{ij} = \frac{1}{2\pi T} \sum Q_n \ln(r)_n \]

- \( s_{ij} \) = drawdown at \((i,j)\) due to extraction
- \( Q_n \) = extraction rate rate at well \( n \)
- \( \ln(r)_n \) = log of separation distance

\[ Z_{est} = f(Q, X, Y, R) \]
Reality Checks - Aquifer

- The estimated transmissivity should correspond with other sources of information - e.g. pumping tests.

and

- The estimated capture zone width should correspond with the estimated transmissivity, pumping rate, and hydraulic gradient.
Capture Zone Width

\[ W = \frac{Q}{K \times B \times i} \]

- \( W \) = capture zone width
- \( K \) = average hydraulic conductivity
- \( Q \) = extraction rate
- \( B \) = saturated thickness
- \( i \) = hydraulic gradient

*Note: Half-width at the recovery well*
Stagnation Point

\[ h = \frac{W}{2 \times \pi} \]

\( h \) = distance of stagnation point down gradient of the recovery well

\( W \) = capture zone width

\( \pi \) = 3.14159...
Reality Checks - Aquifer

\[ W = \frac{Q}{KBi} \]

Stagnation \[= \frac{w}{2\pi} \]

Half-width \[= \frac{W}{2} \]
Reality Checks - Data

- Do the data meet the assumptions
- Are there ‘outliers’ that require additional explanation
  - Jacknifing
  - Single-point cross-validation
Particle Tracking

Build directly from the kriging routine

- Once the kriging matrix has been constructed the kriging weights that pertain to the estimation of any point are estimable.
- The benefit of this approach is that the gradient and seepage velocity are calculated on the basis of the kriging estimate exactly at the location of the particle.
Particle Tracking
Particle Tracking
Particle Tracking
Particle Tracking
Example Application

Cape Cod, MA.

- Described in the paper by Tonkin and Larson, 2002.

Cape Cod

- CS-10 In-Plume System
- Mass recovery objective
- 5 extraction wells
- Peripheral infiltration galleries
- About 40 monitoring wells
- *Aim to infer if capture is developing*
Cape Cod - CS-10
CS10 - Zoom
Cape Cod - No Trend

\[ Z_{\text{est}} = f(R) \]
Cape Cod - Linear Trend

\[ Z_{\text{est}} = f(X, Y, R) \]
Cape Cod - Linear-log Trend

\[ Z_{est} = f(Q, X, Y, R) \]
Cape Cod - Comparison

Linear drift

Linear-log drift
NOTE – estimated T was 56,000 ft²/d; from pump tests – 35,000 – 60,000 ft²/d
Cape Cod

Time of Travel (Days)
- 0 to 100
- 100 to 200
- 200 to 300
- 300 to 400
- 400 to 499
Profile Example

Software

- Based on GSLIB program KT3D.
- GUI drives KT3D_L1 including particle tracking.
- Surfer™ or Rockware™
Rockworks
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